Corrigé révisions - séance 7

Exercice 1 (ecricome 2003)
$f(x) = \dfrac{2}{(1+x)^3} = 2(1+x)^{-3} = 2 \times \underbrace{1}_{u'(x)} \times \underbrace{(1+x)^{-3}}_{u(x)^{-3}}$.

Donc $F(x) = 2 \times \dfrac{u(x)^{-2}}{-2} = -\dfrac{1}{(1+x)^2}$.

$\displaystyle\int_0^A f(x)dx = \left[-\frac{1}{(1+x)^2}\right]_0^A = -\frac{1}{(1+A)^2} + 1$.

$\displaystyle\int_0^{+\infty} f(x)dx = \lim_{A\to+\infty} \int_0^A f(x)dx = 1$.

Exercice 2 (ecricome 2004-2006 et edhec 2001)
$f(x) = xe^{-\frac{x^2}{2}} = (-1) \times \underbrace{(-x)}_{u'(x)} \times \underbrace{e^{-\frac{x^2}{2}}}_{e^{u(x)}}$.

Donc $F(x) = -e^{-\frac{x^2}{2}}$.

$\displaystyle\int_0^A f(x)dx = \left[-e^{-\frac{x^2}{2}}\right]_0^A = -e^{-\frac{A^2}{2}} + 1$.

$\displaystyle\int_0^{+\infty} f(x)dx = \lim_{A\to+\infty} \int_0^A f(x)dx = 1$.

Exercice 3 (ecricome 2008)
$f(x) = 2x$.

Donc $F(x) = 2 \times \dfrac{x^2}{2} = x^2$.

$\displaystyle\int_0^1 f(x)dx = \left[x^2\right]_0^1 = 1$.

Exercice 4 (edhec 2003)
$f(x) = nx^{n-1}$.

Donc $F(x) = n \times \dfrac{x^n}{n} = x^n$.

$\displaystyle\int_0^1 f(x)dx = \left[x^n\right]_0^1 = 1$.

Exercice 5 (ecricome 2019)
$f(x) = \dfrac{1}{x^3} = x^{-3}$.

Donc $F(x) = \dfrac{x^{-2}}{-2} = -\dfrac{1}{2x^2}$.

$\displaystyle\int_1^A f(x)dx = \left[-\frac{1}{2x^2}\right]_1^A = -\frac{1}{2A^2} + \frac{1}{2}$.

$\displaystyle\int_1^{+\infty} f(x)dx = \lim_{A\to+\infty} \int_1^A f(x)dx = \frac{1}{2}$.

Exercice 6 (ecricome 2020)

$f(x) = \dfrac{1}{x^n} = x^{-n}$.

Donc $F(x) = \dfrac{x^{-n+1}}{-n+1} = \dfrac{1}{(-n+1)x^{n-1}}$.

$\displaystyle\int_1^A f(x)dx = \left[\frac{1}{(-n+1)x^{n-1}}\right]_1^A = \frac{1}{(-n+1)A^{n-1}} - \frac{1}{-n+1}$.

$\displaystyle\int_1^{+\infty} f(x)dx = \lim_{A\to+\infty}\int_1^A f(x)dx = \frac{1}{n-1}$.

Exercice 7 (edhec 2005 et eml 2015)

$f(x) = e^{-\lambda x} = \dfrac{-1}{\lambda} \times \underbrace{-\lambda}_{u'(x)} \times \underbrace{e^{-\lambda x}}_{e^{u(x)}}$.

Donc $F(x) = \dfrac{-1}{\lambda} \times e^{-\lambda x}$.

$\displaystyle\int_0^A f(x)dx = \left[\frac{-1}{\lambda}\times e^{-\lambda x}\right]_0^A = \frac{-1}{\lambda}\times e^{-\lambda A} + \frac{1}{\lambda}$.

$\displaystyle\int_0^{+\infty} f(x)dx = \lim_{A\to+\infty}\int_0^A f(x)dx = \frac{1}{\lambda}$.

Exercice 8 (edhec 2008 et eml 2005)

$f(x) = \dfrac{1}{(1+x)^2} = (1+x)^{-2} = \underbrace{1}_{u'(x)} \times \underbrace{(1+x)^{-2}}_{u(x)^{-2}}$.

Donc $F(x) = \dfrac{u(x)^{-1}}{-1} = -\dfrac{1}{1+x}$.

$\displaystyle\int_0^A f(x)dx = \left[-\frac{1}{1+x}\right]_0^A = -\frac{1}{1+A} + 1$.

$\displaystyle\int_0^{+\infty} f(x)dx = \lim_{A\to+\infty}\int_0^A f(x)dx = 1$.

Exercice 9 (edhec 2019)

$f(x) = \dfrac{1}{2x^2} = \dfrac{1}{2} \times x^{-2}$.

Donc $F(x) = \dfrac{1}{2} \times \dfrac{x^{-1}}{-1} = -\dfrac{1}{2x}$.

$\displaystyle\int_1^A f(x)dx = \left[-\frac{1}{2x}\right]_1^A = -\frac{1}{2A} + \frac{1}{2}$.

$\displaystyle\int_1^{+\infty} f(x)dx = \lim_{A\to+\infty}\int_1^A f(x)dx = \frac{1}{2}$.

Exercice 10 (eml 2001 et eml 2007)
$f(x) = e^{-x} = (-1) \times \underbrace{-1}_{u'(x)} \times \underbrace{e^{-x}}_{e^{u(x)}}$.

Donc $F(x) = (-1) \times e^{-x} = -e^{-x}$.

$\int_0^A f(x)dx = \left[-e^{-x}\right]_0^A = -e^{-A} + 1$.

$\int_0^{+\infty} f(x)dx = \lim_{A \to +\infty} \int_0^A f(x)dx = 1$.

Exercice 11 (ecricome 2003)
$f(x) = \frac{1}{x\sqrt{2x}} = \frac{1}{\sqrt{2}} \times x^{-3/2}$.

Donc $F(x) = \frac{1}{\sqrt{2}} \times \frac{x^{-1/2}}{-1/2} = -\frac{2}{\sqrt{2}\sqrt{x}} = -\frac{\sqrt{2}}{\sqrt{x}}$.

$\int_2^A f(x)dx = \left[-\frac{\sqrt{2}}{\sqrt{x}}\right]_2^A = -\frac{\sqrt{2}}{\sqrt{A}} + 1$.

$\int_2^{+\infty} f(x)dx = \lim_{A \to +\infty} \int_2^A f(x)dx = 1$.

Exercice 12 (eml 2016)
$f(x) = \frac{e^{-x}}{(1+e^{-x})^2} = e^{-x}\left(1+e^{-x}\right)^{-2} = (-1) \times \underbrace{-e^{-x}}_{u'(x)} \times \underbrace{\left(1+e^{-x}\right)^{-2}}_{u(x)^{-2}}$.

Donc $F(x) = (-1) \times \frac{u(x)^{-1}}{-1} = \frac{1}{1+e^{-x}}$.

$\int_0^A f(x)dx = \left[\frac{1}{1+e^{-x}}\right]_0^A = \frac{1}{1+e^{-A}} - \frac{1}{2}$.

$\int_0^{+\infty} f(x)dx = \lim_{A \to +\infty} \int_0^A f(x)dx = 1 - \frac{1}{2} = \frac{1}{2}$.

$\int_B^0 f(x)dx = \left[\frac{1}{1+e^{-x}}\right]_B^0 = \frac{1}{2} - \frac{1}{1+e^{-B}}$.

$\int_{-\infty}^0 f(x)dx = \lim_{B \to -\infty} \int_B^0 f(x)dx = \frac{1}{2} - 0 = \frac{1}{2}$.

Donc $\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^0 f(x)dx + \int_0^{+\infty} f(x)dx = \frac{1}{2} + \frac{1}{2} = 1$.

Exercice 13 (edhec 2019)
$f(x) = \frac{1}{\theta x^{1+\frac{1}{\theta}}} = \frac{1}{\theta} \times x^{-1-\frac{1}{\theta}}$.

Donc $F(x) = \frac{1}{\theta} \times \frac{x^{-\frac{1}{\theta}}}{-\frac{1}{\theta}} = -x^{-\frac{1}{\theta}}$.

$\int_1^A f(x)dx = \left[-x^{-\frac{1}{\theta}}\right]_1^A = -x^{-\frac{1}{A}} + 1$.

$\int_1^{+\infty} f(x)dx = \lim_{A \to +\infty} \int_1^A f(x)dx = 1$.